CHARACTERISTICS OF CARBON BLACK DUST AS A LARGE-SCALE TROPOSPHERIC HEAT SOURCE

by
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ABSTRACT

This paper evaluates the radiation properties of clouds consisting of carbon black particles in aerosol form spread artificially in the atmosphere to absorb solar radiation and hence to create an atmospheric heat source for possible large-scale weather modification. Properties of carbon black are discussed. A method for estimating absorption of solar radiation by clouds developed by Korb and Möller (1962) is applied to study solar absorption and scattering of carbon black dust clouds. The very high energy gain to weight and cost is discussed. The economics of dispersal is also touched on.

I. INTRODUCTION

The potential use of carbon black dust as an atmospheric heat source has been known for years. The amount of solar energy which can be absorbed by carbon black and transmitted to the air in one day is about 6000 times greater than the amount of heat obtainable by burning a similar mass of coal, making carbon an attractive energy source for weather modification. To date, research on this subject has been concerned primarily with fog and natural cloud dissipation by using large carbon dust concentrations in relatively small areas to evaporate clouds (Downie and Smith 1958, Fenn and Oser 1962, Fenn 1964, Smith et al. 1959, Van Straten et al. 1958, Wexler 1958). The application of this approach has been limited by the large amounts of heat required to evaporate meaningful amounts of moisture and the reduced solar radiation available inside the cloud. Such problems are to be expected when attempting weather modification through "brute force" techniques, where one operates directly against the very high energy levels needed for evaporation (i.e., 600 cal/gram).

This paper is concerned with the potential use of carbon black as a clear air heat source by spreading particles over a large area in order to trigger beneficial mesoscale or synoptic scale flow changes. The radiation properties of carbon clouds of large horizontal extent compared to their vertical depth are studied incorporating the effects of water vapor and planetary surface albedo. Quantitative results are obtained for several cloud models.

Previous Carbon Black Experiments. Several experiments using carbon black as a heat source to dissipate fog and small cumulus clouds and to form small clouds in clear air have been performed.

A) The Naval Research Laboratory (1958) seeded 8 cumulus clouds with 1-1/2 lbs to 6 lbs of carbon black in July, 1958. All of the clouds dissipated to some extent, but observation and instrumentation capabilities were insufficient to establish a definite causal relationship. In addition, clear air at the approximate level of existing cumulus cloud bases was seeded on 5 runs during the same series of tests. Small clouds were observed to form in all cases. Once again it was impossible to establish definite causal relationships. The overall feeling of the test group was that the carbon black did seem to help dissipate existing clouds and form small ones in clear air but the natural variability of cumulus clouds and the inadequacy of monitoring techniques prohibited any conclusive results.

Laboratory tests by the Naval Research Laboratory in 1958 showed that carbon black did increase dissipation rates of artificially created fogs in cloud chambers which were subjected to heat lamps. However, neither the dissipation mechanism nor the radiative properties of carbon black were quantitatively well established.

B) The Geophysics Research Directorate made 18 runs seeding small clouds and clear air in October, 1958-April, 1959. Carbon amounts from 1-1/2 lbs to 5 lbs per mission were used. Observed results were less successful than those observed earlier by the Naval

Research Laboratory. A few clouds dissipated, but others did not. Clear air seeding produced no obvious results although a few small clouds occasionally formed in the test areas. The test personnel concluded that no definite effects of carbon black on clouds could be substantiated through the test results.

In general, these early experiments with carbon black suffered from three major shortcomings. The existing knowledge of the radiative properties of carbon black was entirely inadequate to provide realistic estimates of the energy processes occurring in the atmosphere. The amounts of carbon used were much too small. Small scale atmospheric circulation effects could easily dissipate any heat absorbed and overpower the effects of the heat accumulation. Finally, adequate observation and instrumentation capabilities to enable conclusive analysis of field test results were not available. Nevertheless, it seems unlikely that all of the observed results can be attributed to natural causes. These and other early efforts have supplied data for planning future experiments using carbon black for weather modification.

II. RADIATIVE PROPERTIES OF CARBON BLACK DUST IN THE ATMOSPHERE

Carbon Black. Carbon black dust consists of fine spherical particles composed of 95-99% pure carbon, the remainder being made up of volatile materials. It is formed by the controlled incomplete combustion of fossil fuels (usually natural gas) according to a variety of processes depending upon the size and purity of the particles desired. Carbon black is commercially available in sizes from .01 μ to .25 μ in diameter. The uniformity of size of any class of particles is generally good. Most carbon blacks can be obtained in quantity for about \$.07 per pound. They are used presently as a coloring pigment and as a rubber reinforcing agent for auto tires (Cabot, Inc., 1949).

The density of the carbon particles is about 2.0 g/cm^3 while the packing densities of the larger particles range from about 0.3 to 0.6 g/cm^3. Small particles tend to clump together during packing and storage forming irregularly shaped particles about an order of magnitude larger in diameter than the original particles. It is felt that this problem can be overcome using present technology and that the particles can be dispersed discretely into the atmosphere. Once the particles have been so dispersed, later agglomeration in the air is negligible due to the relatively small particle sizes and low particle concentrations that are proposed for use. The high radiation absorptivity, and low heat capacity (about .125 cal/g^oC) of carbon black make it an ideal agent for interception of solar radiation and transfer

of this heat to the surroundings by conduction. These properties are discussed in more detail later.

Radiation Characteristics of Carbon Black. The absorption (σ_A), scattering (σ_S) and extinction (σ_E) cross sections of spherical carbon particles (complex refractive index 1.59 - .66 i) were computed by Fenn (1962, 1964) according to Mie scattering theory. These coefficients are defined as the ratios between the equivalent areas with which particles absorb, scatter, and extinct light and the actual geometric cross section. They are functions of the refractive index of carbon, particle size and the wave length of the affected light. The coefficients are related as shown in equation (1).

$$\sigma_E = \sigma_A + \sigma_S = \text{extinction cross section} \qquad (1)$$

Values of these cross sections for carbon black particles are plotted as functions of the size paramter $\alpha = \frac{2\pi r}{\lambda}$ in figure 1. Note that the absorption cross section is considerably larger than one for values of α larger than one. Carbon black is commercially available, and the particles so obtained are virtually all spherical and of relatively uniform size and composition. The relevance of the Mie scattering theory to light interference by carbon particles therefore seems to be reasonable. The practical problems of distributing small carbon particles in an aerosol are discussed later. They have been considered in general form and are not felt to be of major proportions.

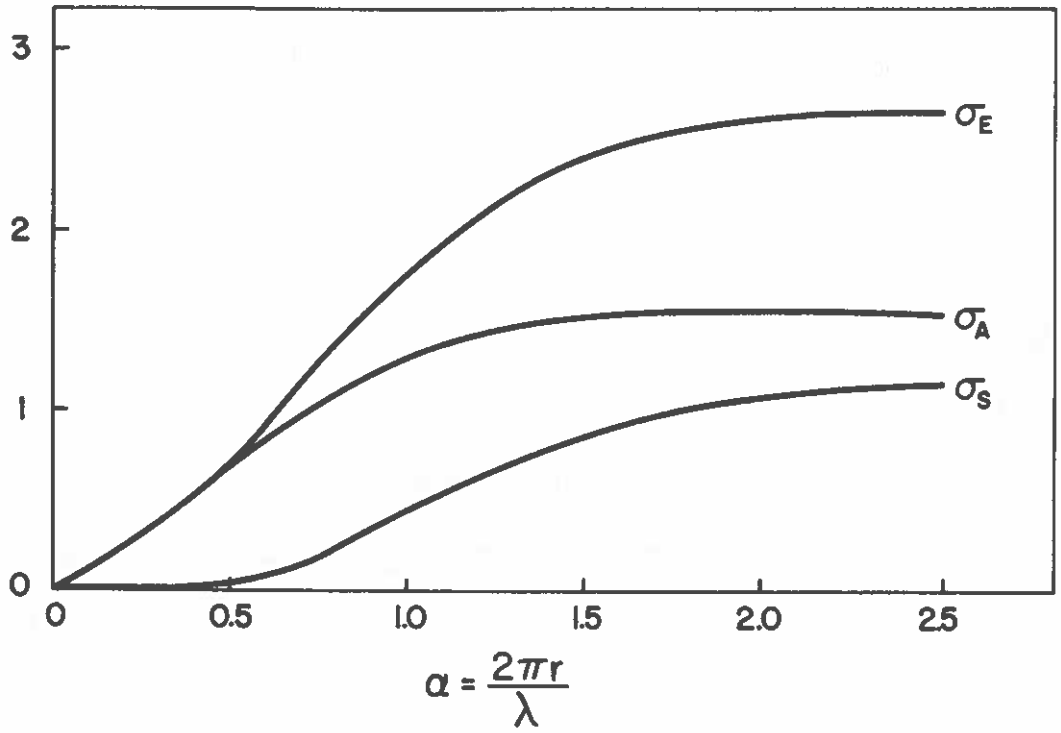

Figure 1. Extinction (σ_E), absorption (σ_A), and scattering (σ_S) cross sections of spherical carbon particles as functions of size parameter (α).

Characteristics of the Carbon Cloud. For this study each carbon dust cloud was assumed to be composed of uniform carbon particles. The clouds are of large horizontal extent compared to their thickness, and each cloud or cloud layer was assumed to be homogeneous. Water vapor contents of the clouds were assigned according to standard atmospheric concentrations depending upon cloud height. All water vapor was assumed to be uncondensed.

The above assumptions were made for convenience of computation. Real situation variations from these assumed values are not felt to be large enough to appreciably alter the results to be shown. Any of the

assumptions can be varied to meet individual case refinement as desired.

Determination of Optimum Particle Size. For economic reasons it is desirable to maximize the amount of radiation absorbed by the carbon particles per unit mass. To do this the optimum particle size must be determined. Since the cloud is of relatively large horizontal extent compared to its height, much of the forward scattered incident radiation will be absorbed due to increased optical path length. Hence, we wish to maximize the extinction coefficient of the cloud per unit mass, rather than the absorption coefficient.

The extinction coefficient of a cloud is given by:

$$K_E = N \cdot \pi r^2 \cdot \sigma_E \qquad (2)$$

where N = number of particles per cm^3

r = radius of particles

σ_E = extinction cross section (a function of α as determined from Mie scattering theory).

Let the volume of a single carbon particle = $V_P = \frac{4}{3} \pi r^3$.

Let the total mass of carbon particles = $M_c = V_T \rho_c$

where ρ_c = density of carbon black (~ 2 gm/cm^3)

V_T = total volume of particles

$$N = \frac{V_T}{V_P} = \frac{M_c}{\rho_c} \cdot \frac{3}{4\pi r^3}. \qquad (3)$$

Then $$K_E = \frac{3}{4} \frac{M_c}{\rho_c} \frac{1}{r} \sigma_E . \tag{4}$$

The size parameter (α) is defined: $\alpha = \frac{2\pi r}{\lambda}$ where λ is the wavelength of radiation. Hence

$$r = \frac{\alpha \lambda}{2\pi} \quad \text{and} \tag{5}$$

$$K_E = \frac{3\pi M_c}{2\rho_c \lambda} \frac{\sigma_E}{\alpha} . \tag{6}$$

For a given mass of carbon per unit volume:

$$\frac{3\pi M_c}{2\rho_c \lambda} = \text{constant}.$$

Hence:

$$K_E = \frac{\sigma_E}{\alpha} \cdot (\text{const.}), \tag{7}$$

$\frac{\sigma_E}{\alpha}$ is plotted in figure 2. The maximum value of $\frac{\sigma_E}{\alpha}$ from this graph will give us maximum value of K_E for any given mass per unit volume. Solving graphically K_E is a maximum at approximately $\alpha = 1.0$.

Although sunlight has an intensity peak at $\lambda = .5\mu$, to maximize the extinction across the entire solar spectrum we shall use the median value of solar wave length which is approximately: $\bar{\lambda} \simeq .72\mu$. This gives an optimum radius of:

$$r = \frac{\alpha \lambda}{2\pi} = \frac{(1.0)(\lambda)}{2\pi} \qquad (8)$$

$$r = .11 \text{ micron } (\mu).$$

For simplicity we shall use particles of $r = .1\mu$ for the remainder of this study. Note from the gradual slope of the $\frac{\sigma_E}{\alpha}$ curve in figure 2 that light extinction per unit mass of carbon is not highly sensitive to particle size changes. Size quality control should not be a crucial problem.

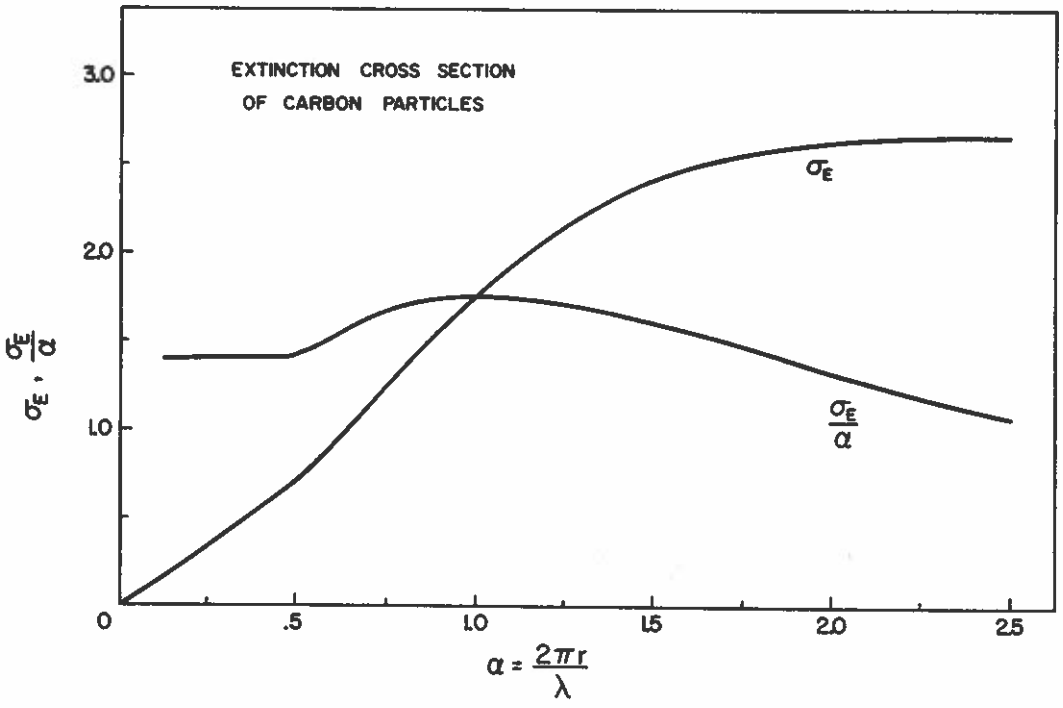

Figure 2.

Division of the Solar Spectrum. To average the general transmission function, the solar spectrum must be divided into finite bands, and average values of the extinction coefficient and optical depth must be determined for each band. These parameters vary rather smoothly with changing wave length. However, water vapor absorption is quite irregular with respect to wave length. It is therefore desirable to choose bands such that each of the absorption bands of water vapor coincide with one of the defined spectral bands. In the spectral bands with no water vapor absorption, water vapor absorption will be zero, and in the bands which coincide with water vapor bands, average values may be determined. (The solar constant value used is 1.95 ly/min.) The spectral divisions used, the water vapor absorption bands, and the solar irradiance (radiant flux incident on a unit area) of each band at the top of the atmosphere are shown in Table 1. Also shown are the values of the absorption (σ_a), scattering (σ_s), and extinction (σ_E) cross sections and the absorption quantity (K_A). Absorption quantity is defined as the ratio of the absorbed light to absorbed plus scattered light as shown in equation (7).

$$K_A = \frac{\text{absorption coefficient of carbon} + \rho_W K_W}{\text{extinction coefficient of carbon} + \rho_W K_W} \qquad (9)$$

where

ρ_W = density water vapor

K_W = absorption coefficient of water vapor,

TABLE 1

DIVISION OF SOLAR SPECTRUM

Region	Wavelength Band ($\Delta\lambda$) (microns)	Water Vapor Band	Incidient Solar Energy $\frac{cal}{cm^2 min}$	Abs. σ_A	Scatt. σ_S	Ext. σ_E	Absorption Quantity K_A
1	.28 - .42	-	.220	1.55	1.03	2.58	.60
2	.42 - .50	-	.232	1.50	.78	2.28	.66
3	.50 - .60	-	.270	1.42	.60	2.02	.70
4	.60 - .70	-	.228	1.30	.44	1.74	.75
5	.70 - .74	(.72μ)	.077	1.20	.33	1.53	.79
6	.74 - .79	-	.089	1.15	.27	1.42	.81
7	.79 - .84	.8μ	.075	1.08	.22	1.30	.84
8	.84 - .86	-	.030	1.03	.20	1.23	.84
9	.86 - .98	.94μ	.143	.95	.15	1.10	.89
10	.98 - 1.05	-	.069	.87	.10	.97	.90
11	1.05 - 1.22	1.13μ	.138	.77	.07	.84	.92
12	1.22 - 1.61	1.40μ	.181	.62	.02	.64	.98
13	1.61 - 2.10	1.87μ	.099	.46	0	.46	1.00
14	2.10 - 2.20	-	.013	.38	0	.38	1.00
15	2.20 - 3.00	2.7μ	.052	.31	0	.31	1.00
16	3.00 - 3.80	3.2μ	.020	.22	0	.22	1.00
17	3.80 - 4.50	-	.008	.19	0	.19	1.00
18	4.50 - 10.00	6.3μ	.012	.14	0	.14	1.00
		Total:	1.95				
		Median:		1.20	.33	1.53	

Method for Solving the Equation of Radiative Transfer Through a Cloud. A method developed by Korb and Möller (1962) was used to solve the general equation of radiative transfer through a cloud (as developed by Chandrasekhar, 1960). The radiation field was broken up into $2 + 2\pi$ fluxes; one directed upward, one downward, and 2π fluxes in the horizontal plane. This breaks the general transfer equation down into a series of linear differential equations which give absorption, transmission, and reflection of radiation as percentages of total incident radiation. These equations are shown in appendix I for the cases where surface albedo is zero and where surface albedo is non zero.

The absorption, reflection, and transmission were determined for each given zenith angle and optical depth for a narrow spectral band, and the results were summed for all spectral bands to give total values for given zenith angle and optical depth. Nine zenith angles from 0° to 80° at 10° intervals were chosen. For simplicity, calculations of daily absorption assumed a 12 hour day with the sun directly overhead at noon. The total absorption and transmission per day for each optical depth was obtained by time averaging the values over the 10 hour 40 minute period with zenith angles less than 80° in computation steps of 10° zenith angle change (40 minutes). Water vapor absorption within the cloud was also included in the computations.

The above radiation model was run assuming no surface reflection and also for various surface albedos by introducing an upward diffuse

radiation field using mean values of the scattering function components. Intensity of the upward scattered field varied with albedo and the amount of light initially transmitted through the cloud.

Solar irradiances at the top of each cloud were reduced using calculations by Dave and Furukawa (1966) to account for ozone (O_3) absorption and molecular scattering above 11 km. Molecular scattering within the cloud was not considered. Absorption by molecular oxygen (O_2) and carbon dioxide (CO_2) is insignificant for the purposes of these calculations and was not taken into account. Effects of natural tropospheric aerosols were not considered in these calculations, but they are discussed later.

For each cloud configuration studied, the total absorption of solar radiation by carbon and water vapor was calculated. The absorption of the equivalent volume at the same altitude due to water vapor alone was then calculated. It was assumed that water vapor in the air around the test cloud would absorb radiation at the rate of the carbon-free air. Since the evaluation of carbon black as a heat source was the object of the study, the net effectiveness of the carbon is the difference between the total solar radiation absorbed by the carbon cloud and the radiation that would be absorbed without the carbon cloud. It was expected that the carbon and water vapor cloud together would absorb less than the sum of the absorptions of a carbon only cloud and a water vapor only cloud due to the redundancy of the absorption characteristics of the two substances at longer wave lengths. This proved to be the case, but the loss in efficiency was very small.

III. RESULTS

Absorption in the Tropical Atmosphere. It is felt that the tropical atmosphere presents the best opportunities for beneficial large scale weather modification. Particular interest is centered on carbon black dust seeding into the tropical boundary layer (1013 mb-950 mb). Absorptions were first computed for clouds containing 4.0 cm and 5.0 cm of precipitable water vapor-(ppw)-(but no carbon) to simulate clear air absorption in the tropical atmosphere. These are the amounts of water vapor found above 950 mb and 1013 mb respectively in a mean tropical cloud cluster. The amounts of solar radiation absorbed per day in clear tropical conditions are shown in Table 2. It is noteworthy that the 5.0 cm ppw atmosphere absorbs only about 5% more radiation per day than does the 4.0 cm ppw atmosphere. This also indicates that 95% of the solar radiation absorbed daily by the 5.0 cm ppw atmosphere is absorbed by the portion of that atmosphere containing the uppermost 4.0 cm. Since so little solar radiation (about 9 ly/day) is absorbed by the lowest 1 cm of ppw, no significant redundancy in absorption by water vapor and carbon would occur in a carbon dust cloud dispersed between 1013 mb and 950 mb in the tropics. Therefore, such a carbon dust cloud can be treated as a dry cloud to a reasonably high degree of accuracy.

Calculations of absorption by a dry carbon cloud in the tropical boundary layer (1013-950 mb) were made using the reduced values of solar energy incident at the 950 mb level to approximate the effects

TABLE 2

SHORT WAVE ABSORPTION BY CLEAN AIR TROPICAL ATMOSPHERE

cm precipitable water	4.00	5.03
Absorption: (ly/day)	153	162

TABLE 3

SHORT WAVE ABSORPTION BY CARBON DUST CLOUDS OF VARIOUS CONCENTRATIONS IN THE TROPICAL BOUNDARY LAYER (1013-950 mb)

Percent Area Coverage:	0	9%	18%	26%	35%	53%	70%
Concentration (Particles/cm^3):	0	5,000	10,000	15,000	20,000	30,000	40,000
Net Absorption: by Carbon (ly/day)	0	111	198	268	325	412	473

of water vapor, ozone, and molecular scattering. A surface albedo of 10% was assumed. Results are shown in Table 3, and net absorption as a function of carbon concentration is shown in figure 3. An increasing loss of efficiency of absorption per unit mass occurs at increasing densities as progressively less solar radiation penetrates to the lower levels of carbon, especially at high zenith angles. Therefore, from an economic point of view it is desirable to cover large areas with low concentrations if the magnitude of heat gain to carbon dust expended are to be maximized. A carbon particle concentration of 10,000 particles/cm^3 represents 18% horizontal area coverage for this cloud model. This would require only 45 kg of carbon black dust per square kilometer of horizontal cloud area.

Absorption by Carbon Dust in the Mid-Latitude Standard Atmosphere. Computations of daily absorption were made for a cloud extending from sea level to 11 km containing 2.8 cm ppw, the approximate mid-latitude standard atmospheric water vapor content, and carbon particle concentrations from zero to 2000 particles/cm^3. It was assumed that such a cloud model might be used to intensify existing broadscale circulations. Total absorptions and net usable absorptions (total absorption minus natural clear air water vapor absorption) per day are shown in Table 4. These absorptions are plotted as functions of cloud density in figure 4. Absorption in langleys per minute rates are plotted as a function of zenith angle for each particle concentration in figure 5. This shows the relative effects of decreasing incident solar

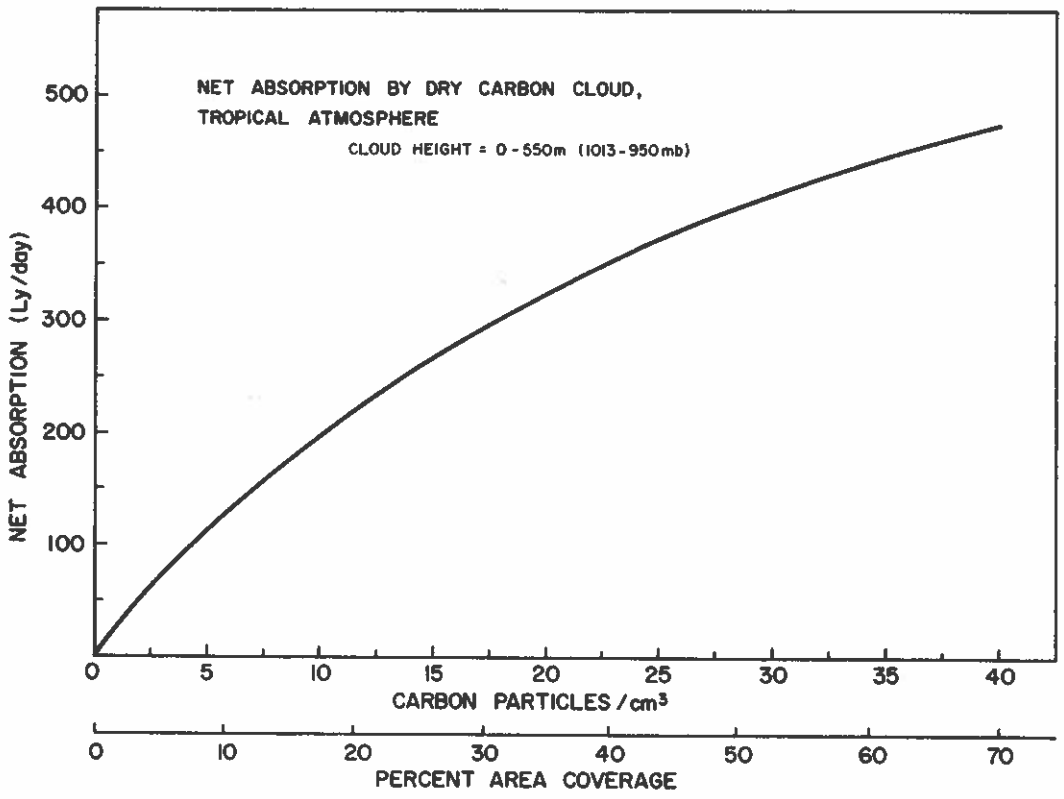

Figure 3.

radiation and increasing optical path length with zenith angle change. It can readily be seen that when dealing with large concentrations of carbon dust, efficiency is considerably lower at zenith angles greater than 60^{o}, which comprises roughly a third of the 12 hour day. Operations requiring large concentrations of carbon black for intense local heating would be most economical during the mid-day hours, while those requiring only moderate concentrations ($N < 500$ particles/cm^3) would be efficient throughout the entire solar day.

Influence of Water Vapor Absorption. To obtain an estimate of the magnitude of the efficiency loss due to the redundant absorption tendencies of carbon and water vapor at higher wave lengths, absorption

TABLE 4

ABSORPTION BY CARBON DUST CLOUD

Cloud Depth 0-11km

Area Coverage	0%	9%	18%	26%	35%	53%	70%
Particles/cm^3	0	250	500	750	1000	1500	2000
Total Absorbed by Cloud	134 ly	247 ly	337 ly	409 ly	469 ly	561 ly	627 ly
Absorbed by Water Vapor in Clear Air	134 ly	134 ly	134 ly	134 ly	134 ly	1341ly	134 ly
Net Usable Absorbed	0	113 ly	203 ly	275 ly	335 ly	427 ly	493 ly

N = particles/cm^3

C = % area Coverage of Carbon

ly lost by competing with water vapor	0	10 ly	20 ly	30 ly	37 ly	50 ly	61 ly

(absorption by carbon cloud with no water vapor-net usable absorbed)

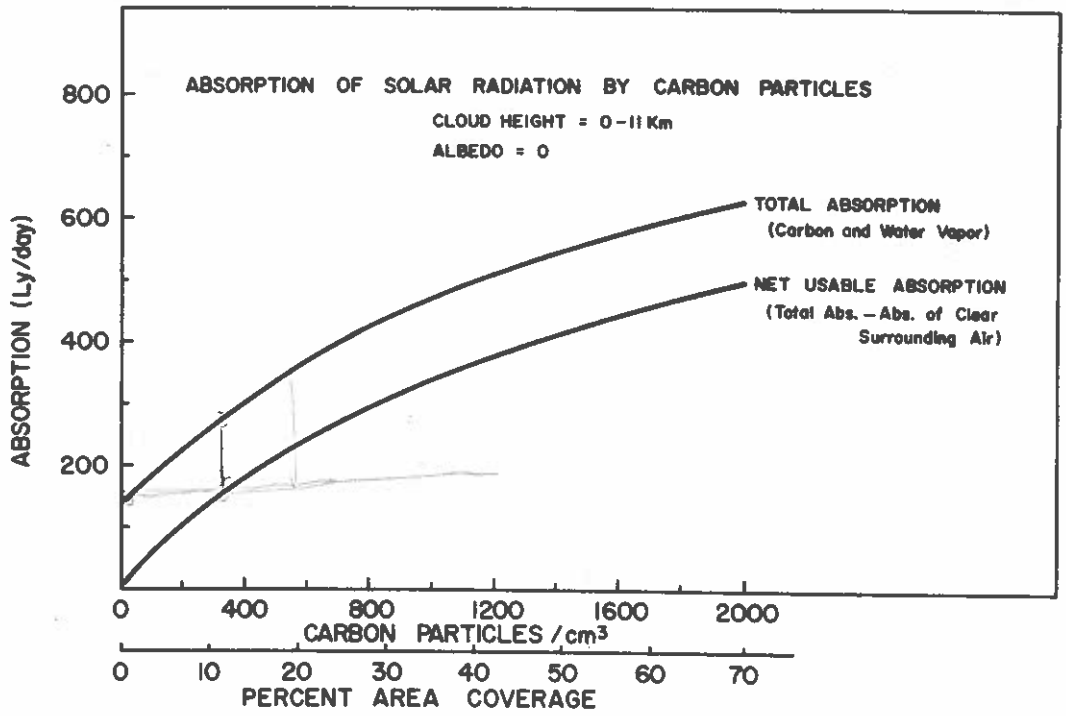

Figure 4.

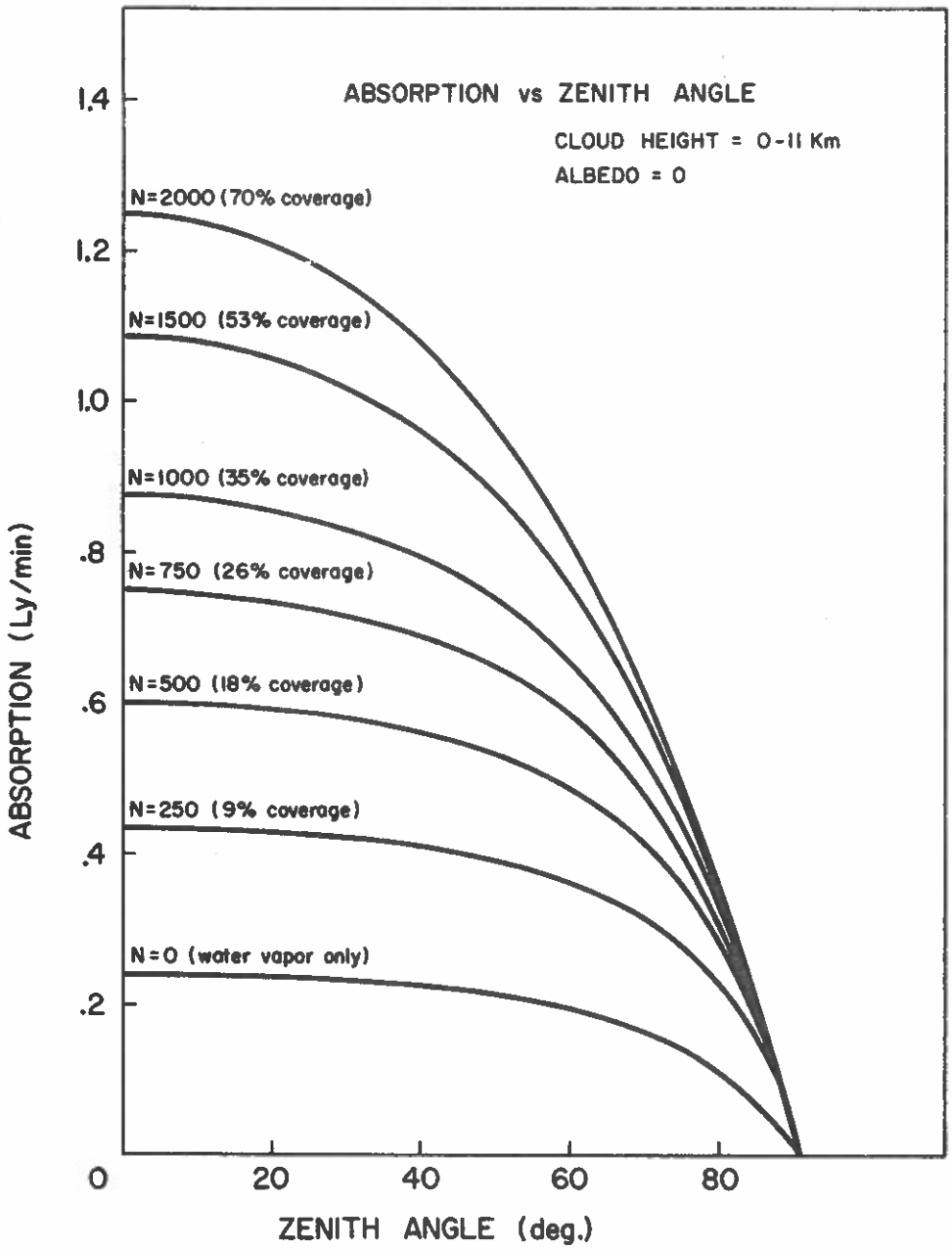

Figure 5.

values of a 0-11 km carbon cloud were calculated assuming water vapor content to be zero. The results are plotted in figure 6 along with the total and net usable absorption potential for the mid-latitude model. The absorption loss increases with particle concentration, and the ratio of lost absorption to net usable absorption increases slightly. The

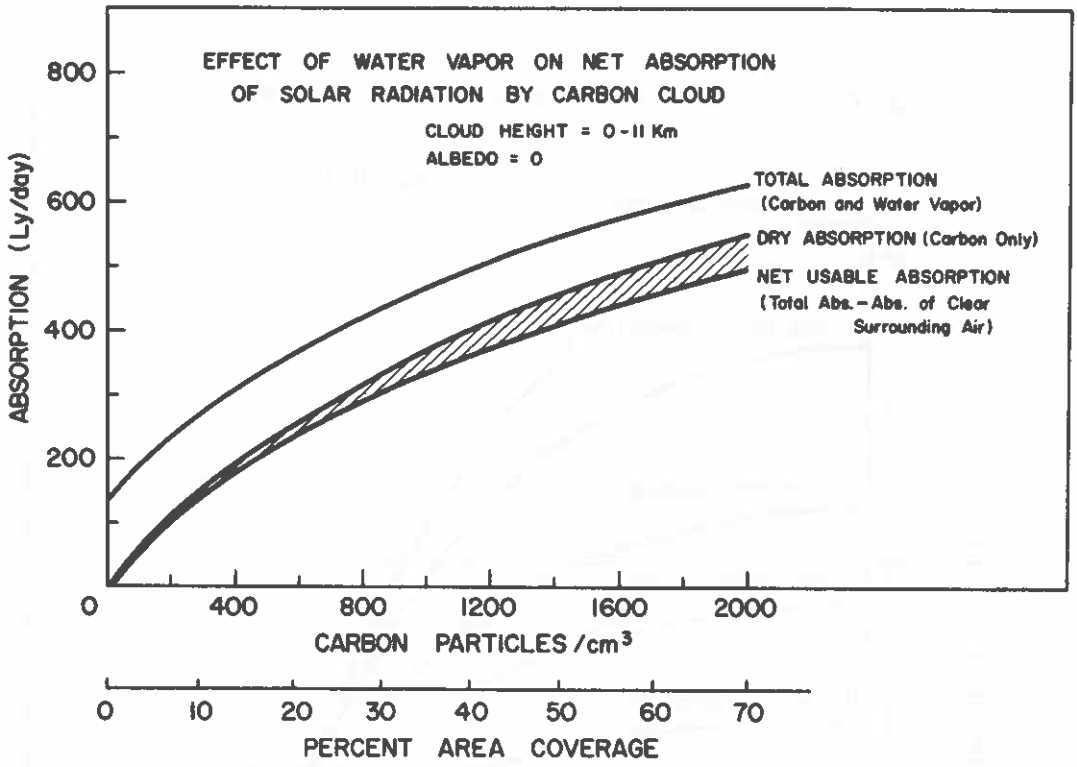

Figure 6.

lost absorption ranges from 0% to about 12% of net usable absorption for the densities tested.

Heating of Air by Carbon. It is necessary to develop the heat budget of carbon particles in air to show that virtually all of the solar radiation absorbed is used to heat the air. Consider the case of a carbon cloud in the tropical boundary layer (1013-950 mb) with a carbon particle concentration of $N = 5000$ particles/cm^3 (9% area coverage). Assume a mean molecular weight of air of $M = 29$ g/mole and a mean layer temperature of $T_o = 22^oC$ (295^oK). The net absorption by the carbon in the layer determined from Table 3 is 111 ly/640 min. The total carbon mass in a cloud column of 550 m with a cross section of 1 cm^2 is:

$$\text{Mass}_{\text{carbon}} = (\text{N per Volume})\,(\text{Volume column})\,(\text{Volume particle})\,(\rho \text{ carbon}) \qquad (10)$$

$$\text{Mass}_{\text{carbon}} = (5000\,\text{cm}^{-3})\,(5.5 \times 10^{4}\,\text{cm}^{3})\,(\tfrac{4}{3}\pi \times 10^{-15}\,\text{cm}^{3})\,(2.0\,\text{g/cm}^{3})$$

$$\text{Mass}_{\text{carbon}} = 2.4 \times 10^{-6}\,\text{g}$$

$\frac{\Delta h}{\Delta t}$ = energy absorbed per unit time

$$\frac{\Delta h}{\Delta t} = \frac{111\,\text{cal}}{(2.4 \times 10^{-6}\,\text{g})\,(640\,\text{min})} = 7.2 \times 10^{4}\,\text{cal/g min}$$

$$\frac{\Delta h}{\Delta t} = C_{\text{carbon}}\,\frac{\Delta T}{\Delta t} \qquad (11)$$

where C_{carbon} = specific heat carbon

$$C_{\text{carbon}} = \frac{.125\,\text{cal}}{^{o}\text{K g}}$$

$$\frac{7.2 \times 10^{4}\,\text{cal}}{\text{g min}} = \frac{.125\,\text{cal}}{^{o}\text{K g}}\,\frac{\Delta T}{\Delta t}$$

$\frac{\Delta T}{\Delta t} = 5.8 \times 10^{5}\,^{o}\text{K/min}$ = rate of temperature increase of particles if no heat were removed.

Since the rate of heat accumulation by the carbon particles is so large, it is reasonable to assume that the particles quickly reach equilibrium with respect to heat transfer with their environments.

The equilibrium heat budget of a single carbon particle can be written:

$$\overline{ABS} = E_o + \dot{R}_o \qquad (12)$$

where $\overline{ABS}$ = mean rate of energy absorption per unit surface area of carbon particles

E_o = rate of heat conduction from particles to air per unit surface area

R_o = radiation emitted by a carbon particle per unit surface area.

Particle Geometry Considerations:

Surface area (1 particle) = $4\pi r^2 = 1.26 \times 10^{-9} cm^2$

This cloud model has $N = 5000$ particles/cm^3 and,

$$z = 5.5 \times 10^4 cm$$

Number particles per 1 cm^2 area of a cloud of 550 m depth is,

$$N \cdot h = 2.75 \times 10^8 \text{ particles.}$$

Surface Area of particles in this 1 cm^2 area cloud column is:

$$\text{Total Surface Area}_{carbon} = N \cdot h \cdot 4\pi r^2 = .35\, cm^2$$

$$\text{Absorption in column} = \frac{111 \text{ ly}}{3.8 \times 10^4 \text{ sec}} = 2.9 \times 10^{-3} \text{ ly/sec}$$

$$\overline{ABS} = \frac{2.9 \times 10^{-3} \text{ cal/sec}}{.35\, cm^2} = 8.3 \times 10^{-3} \text{ cal/cm}^2 \text{ sec.}$$

Because the surface area to mass of the particles is so large, the rate of heat transfer per unit surface area is very small.

Conductive Energy Transfer From Particles:

E_o = conductive energy transfer. This can be written (Dushman, 1949),

$$E_o = \nu \cdot 2K \cdot \alpha \, (T_s - T_o) \quad , \tag{13}$$

where ν = number of air molecules striking one cm^2 surface carbon per second

$K = 1.38 \times 10^{-23}$ joules/^oK (Boltzmann's constant)

α = accommodation coefficient of carbon black

T_s = temperature of surface of carbon particles

T_o = ambient temperature (295^oK).

From Dushman's (1949) analysis of the rate at which molecules strike a surface:

$$\nu = \frac{(3.5 \times 10^{22})(P_{mm})}{(MT_o)^{1/2}} \tag{14}$$

where P_{mm} = 710 mm = atmospheric pressure in mm of H_g

M = mol. wt. air = 29 g/mole

$T_o = 295^oK$

$$\nu = \frac{2.5 \times 10^{25}}{93} = 2.7 \times 10^{23} \text{ molecules/cm}^2\text{sec}$$

$$E_o = (2.7 \times 10^{23}/\text{cm}^2\text{sec})(2)(1.38 \times 10^{-23} \text{ joules}/^oK)$$

$$(\tfrac{1}{4.2} \text{ cal/joule})(\alpha)(T_s - 295^oK) \tag{15}$$

$$E_o = (1.8)\,\alpha\,(T_s - 295^oK)\ cal/cm^2 sec\,. \qquad (16)$$

The accomodation coefficient (α) of carbon black in air should closely approximate that of a blackened plate, as published in Dushman (1949). Therefore, it is reasonable to assume that the approximate value of α will be : $0.9 \leq \alpha \leq 1.0$.

Treating the carbon particles as gray body absorbers and emiters, the long wave energy radiated by the carbon particles may be expressed:

$$R_o = E\sigma T_s^4 \qquad (17)$$

where E = emissivity of carbon particles
σ = Stefan Boltzmann Constant = 1.35×10^{-12} cal/cm^2 ^{o}Ksec.

The wavelength of maximum emission (λ_m) can be computed from Wien's displacement law:

$$\lambda_m = \frac{2897^oK\,\mu}{T_s}\,. \qquad (18)$$

Assuming that T_s does not differ greatly from T_o, the approximate value of λ_m is:

$$\lambda_m = \frac{2897^oK\,\mu}{295^oK} = 9.8\,\mu.$$

By definition, the absorption cross section (σ_A) of carbon particles equals the absorptivity of the particles. For values of σ_A less than 1, the absorption cross section approximately equals the emissivity (E). The mean absorption cross section ($\overline{\sigma_{ABS}}$) is the value of the absorption cross section corresponding to $\lambda_m = 9.8\ \mu$ and can be determined from figure 1:

$$E = \overline{\sigma_{ABS}} = .05$$

$$R_o = (.05)(1.35 \times 10^{-12}\ \text{cal/cm}^2\text{sec}^{\circ}\text{K})(T_s^4)$$

$$= (6.8 \times 10^{-14}\ \frac{\text{cal}}{\text{cm}^2\text{sec}^{\circ}\text{K}^4})\ T_s^4.$$

The energy budget of the carbon particles in terms of particle surface area can now be written:

$$\overline{ABS} = E_o + R_o$$

$$8.3 \times 10^{-3}\ \text{cal/cm}^2\text{sec} = (1.8)(\alpha)(T_s - 295^{\circ}\text{K})^{\circ}\text{K cal/cm}^2\text{sec} + (6.8 \times 10^{-14}\ \text{cal/cm}^2\text{sec}^{\circ}\text{K}^4)(T_s)^4 \qquad (19)$$

where $0.9 < \alpha < 1.0$.

At the initial state of carbon dispersal ($T_s = T_o = 295^{\circ}\text{K}$):

$$E_o = 0$$

$$R_o = (6.8 \times 10^{-14})(295)^4 \text{ cal/cm}^2\text{sec} = .52 \times 10^{-3} \text{ cal/cm}^2\text{sec.}$$

At equilibrium R_o is relatively unchanged from its initial state value. Therefore, T_s can be estimated from equation (19) as

$$T_s = (T_o + 4.8 \times 10^{-3})^{\circ}K$$

$$= 295.005\ ^{\circ}K$$

$$E_o = 7.8 \times 10^{-3} \text{ cal/cm}^2\text{ sec}$$

$$R_o = .52 \times 10^{-3} \text{ cal/cm}^2\text{ sec}$$

and $$\frac{E_o}{R_o} \simeq 15.$$

It is clear that most of the heat absorbed by the carbon is conducted to the air (94%), while only 6% is emitted as longwave radiation. At higher altitudes higher carbon dust absorption efficiences and lower ambient air temperatures result in still larger ratios of $\frac{E_o}{R_o}$. Therefore, it is reasonable to assume that most of the solar radiation absorbed by carbon black is transferred to the surrounding air as heat. In addition, much of the longwave radiation emitted by the carbon particles will be reabsorbed by other carbon particles and by water vapor within the dust cloud, especially for moist clouds in the tropical

boundary layer model discussed above. To fully account for all this reabsorption, the long wave radiant flux divergence of the entire cloud must be calculated.

Long Wave Radiation Loss. To develop a complete heat budget of a carbon black cloud, it is necessary to consider the loss of absorbed radiation due to vertical long wave radiant flux divergence. Korb and Möller (1962) calculated values of long wave flux divergence of various carbon clouds for a 24 hour day. They considered CO_2 and water vapor to be selective absorbers and the carbon particles to be gray absorbers. Standard mid-latitude atmospheric values of water vapor concentrations (figure 7) were used. Results are shown in Table 5. The difference between flux loss from a cloud consisting of carbon, water vapor and CO_2 and a similarly dimensional cloud consisting only of water vapor and CO_2 represents the loss of effective heating by the carbon black. In the most extreme case, the 1-6 K_m cloud with a carbon concentration of 1×10^4 particles per cm^3, the effective loss is only 10.3 ly which represents only about 1-2% of the total solar radiation absorbed in one day by such a cloud. At very high altitudes (the 10-11 K_m cloud) there is a slight relative gain of long wave radiation by the cloud. Thus, long wave flux should not have a significant effect on the net radiative energy gain of carbon seeded air. It is assumed, therefore, that virtually all radiation absorbed by the cloud will be directly converted into heat within the absorption layer.

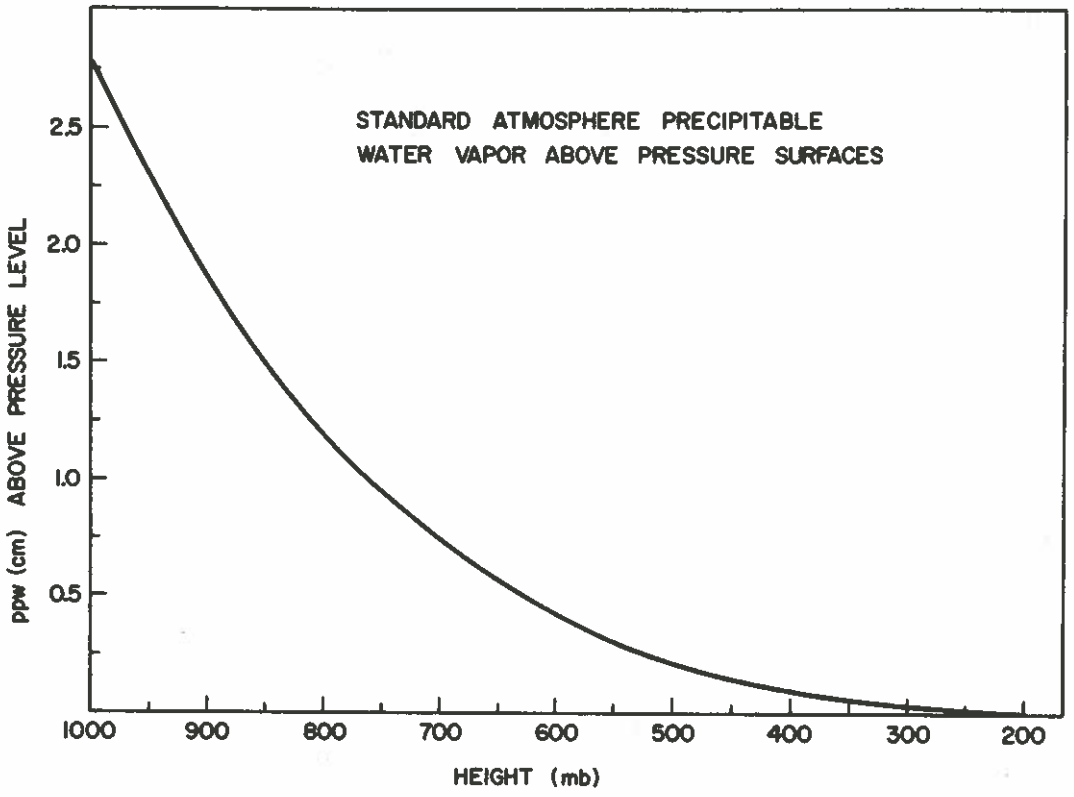

Figure 7.

Influence of Surface Albedo. The effects of varying surface albedo upon total daily absorption were taken into account by applying the upward diffuse radiation field described earlier to the mid-latitude cloud model. The effects are the same for the tropical boundary layer cloud. Computations were made for surface albedo values (A_s) of 0%, 10%, 20%, 30% and 50%. Total and net usable absorption values are shown in Table 6, gains in absorption due to surface albedo are shown in Table 7, and the net absorption values are plotted as functions of particle concentration in figure 8. The increase in net absorption due to surface albedo is greatest between concentrations of about 500 and 1500 particles/cm^3 peaking at about 1000 particles/

TABLE 5

LONG WAVE RADITION FLUX DIVERGENCE LOSS BY VARIOUS AEROSOL CLOUD MODELS (Korb and Moeller, 1962)

	Long Wave Flux Divergence (1y/24hr)				
Cloud Height	(1) H_2O,CO_2 Cloud	(2) H_2O,CO_2,Carbon ($N=10^3 cm^{-3}$) Cloud	[(2) - (1)] Net Loss by Carbon	(3) H_2O,CO_2,Carbon ($N=10^4 cm^{-3}$) Cloud	[(3) - (1)] Net Loss by Carbon
1-2km	-33.7	-34.3	-0.6	-38.6	-4.9
5-6km	-23.3	-23.3	0.0	-24.9	-1.6
1-6km	-126.9	-128.0	-1.1	-137.2	-10.3
10-11km	-14.3	-14.1	+.02	-11.8	+2.5

(Negative values indicate loss)

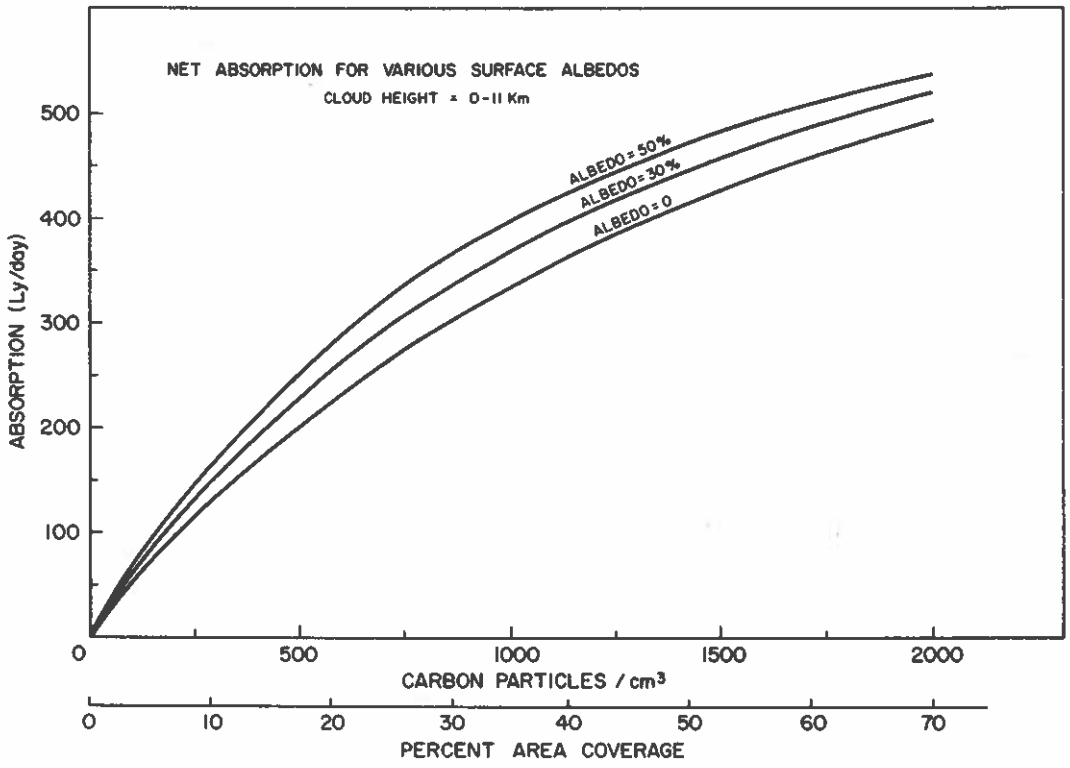

Figure 8.

cm^3 (35% area coverage). It decreases with higher concentrations due to greater initial interception of light and hence lower reflected values. Even with a 50% surface albedo increase net absorption is changed by only 15% or less. Therefore, although the effects upon absorption are not negligible, they are low enough to be safely neglected in many situations without greatly affecting results.

Absorption Distribution in the Vertical. For application of this study to actual operations, it is necessary to know how the absorbed radiation is distributed vertically through the cloud. To obtain an approximation of this a 4 km thick cloud was divided into four directly adjacent 1 km thick layer clouds. For each carbon particle concentration, each cloud layer was computed, the top layer first, the top plus the second cloud next, and so forth. By simple subtraction the

TABLE 6

ABSORPTION CALCULATED INCLUDING SURFACE ALBEDOS

Albedo (As) = 0

C =	0%	9%	18%	26%	35%	53%	70%
	0	250	500	750	1000	1500	2000
TOTAL ABS.	134 ly	247	337	409	469	561	627
W. VAPOR ABS.	134	-	-	-	-	-	-
NET ABS.	0	113	203	275	335	427	493

As = 10%

C =	0%	9%	18%	26%	35%	53%	70%
N =	0	250	500	750	1000	1500	2000
TOTAL ABS.	140	259	352	426	487	578	642
W. VAPOR ABS.	140	-	-	-	-	-	-
NET ABS.	0	119	212	286	347	438	502

As = 20%

C =	0%	9%	18%	26%	35%	53%	70%
N =	0	250	500	750	1000	1500	2000
TOTAL ABS.	145	271	368	443	504	594	663
W. VAPOR ABS.	145	-	-	-	-	-	-
NET ABS.	0	126	223	298	359	449	518

TABLE 6 (cont)

ABSORPTION CALCULATED INCLUDING SURFACE ALBEDOS

As = 30%

C =	0%	9%	18%	26%	35%	53%	70%
N =	0	250	500	750	1000	1500	2000
TOTAL ABS.	151	283	383	460	522	611	671
W. VAPOR ABS	151	-	-	-	-	-	-
NET ABS.	0	132	232	309	371	460	520

As = 50%

C =	0%	9%	18%	26%	35%	53%	70%
N =	0	250	500	750	1000	1500	2000
TOTAL ABS.	162	308	414	498	551	643	699
W. VAPOR ABS	162	-	-	-	-	-	-
NET ABS.	0	146	252	336	396	483	539

amount of solar radiation absorbed in each layer was determined for this homogeneous cloud model. This cloud (cloud Model #2) was located between 200 mb and 100 mb. For simplicity and with previous demonstration of its small effect, water vapor absorption was neglected. Surface albedo was also neglected. In this calculation the upward diffuse radiation field would have been reduced below values in the previous cloud model due to water vapor absorption of both the downward transmitted and the upward reflected radiation. Absorptions in each layer are shown in Table 8. Total absorption is greatest in the upper layer and decreases in each succeeding layer as the amount of light incident at the top of each layer becomes less. As particle concentrations

TABLE 7

GAIN IN NET USABLE ABSORPTION DUE TO SURFACE ALBEDO
(1y/10hr40min)

Albedo	C N	0% 0	9% 250	18% 500	26% 750	35% 1000	53% 1500	70% 2000
0		0	-	-	-	-	-	-
10		0	6	9	11	12	11	9
20		0	13	20	23	24	22	25
30		0	19	29	34	36	33	27
50		0	33	44	61	61	56	44

increase, the percentage of absorbed light which is absorbed in the upper layer increases sharply. Therefore, if vertically homogeneous heating is required, the particles must be distributed with lowest concentrations at the top increasing to the heaviest concentrations at the bottom of the cloud.

Particle Size Distribution in the Vertical. It was felt that due to relatively high extinction of light in the shorter wavelengths, the light incident upon lower layers of a carbon cloud would exhibit longer median wavelengths than would light incident upon upper layers. A significant increase in median wavelength would indicate that larger particles would be required to maintain optimum absorption per unit mass as determined from figure 2. The four layer cloud model from 200 mb to 100 mb was used to evaluate this effect. For each of 3 different carbon particle concentrations, light incident upon the top of each layer was

TABLE 8

ABSORPTION (ly/cm^3), 10hr40min) IN EACH CLOUD LAYER

C(%)	50%	126%	252%
Cloud Levels N($\frac{Part}{cm^3}$)	4000	10,000	20,000
1.	171	349	528
2.	126	179	168
3.	96	164	69
4.	75	64	34
TOTAL (100-200 mb)	468	696	799

estimated by subtracting previously absorbed light from initial incident light intensities for each spectral region. The median wavelength of light incident upon each layer was calculated.

Results for carbon particle concentrations of 4,000, 10,000, and 20,000 particles/cm^3 are shown in Tables 9, 10 and 11, respectively. The expected increase in wavelengths of solar light at lower levels does occur. In the most extreme case (N = 20,000 particles/cm^3) the increase in wavelength of incident light from the highest to the lowest of the 4 layers was 76%, but this is a higher concentration of carbon than would probably be used in most types of applications. For the two lower concentrations wavelengths varied only 14% (N = 4,000 particles/cm^3) and 39% (N = 10,000 particles/cm^3). In the lower parts of the troposphere, water vapor absorption in the longer wavelengths would reduce the amounts of the wavelength increases. From figure 3 it was determined that maximum absorption per unit mass would occur at

TABLE 9

INTENSITIES AND MEDIAN WAVELENGTHS OF INCIDENT SOLAR RADIATION ON SUCCESSIVE LAYERS OF A CARBON CLOUD (N = 4,000 PARTICLES/cm^3)

(Cloud Height = 200-100 mb) (Zenith Angle = 0^o)

Spectral Region	Wave Length (μ)	Incident Intensities (ly/min) Layer 1	Layer 2	Layer 3	Layer 4
1.	.28-.42	.213	.165	.129	.100
2.	.42-.50	.231	.183	.144	.114
3.	.50-.60	.263	.212	.171	.138
4.	.60-.70	.227	.187	.155	.128
5.	.70-.74	.077	.065	.055	.046
6.	.74-.79	.089	.076	.064	.055
7.	.79-.84	.075	.065	.056	.048
8.	.84-.86	.030	.026	.023	.020
9.	.86-.98	.143	.126	.110	.097
10.	.98-1.05	.069	.061	.055	.049
11.	1.05-1.22	.138	.125	.113	.102
12.	1.22-1.61	.181	.167	.154	.143
13.	1.61-2.10	.099	.093	.088	.083
14.	2.10-2.20	.013	.012	.012	.011
15.	2.20-3.00	.052	.050	.048	.046
16.	3.00-3.80	.020	.019	.019	.018
17.	3.80-4.50	.008	.008	.008	.007
18	4.50-10.00	.012	.012	.012	.011
Median Wavelength =		$\bar{\lambda}$ = .72μ	$\bar{\lambda}$ = .74μ	$\bar{\lambda}$ = .78μ	$\bar{\lambda}$ = .82μ

TABLE 10

INTENSITIES AND MEDIAN WAVELENGTHS OF INCIDENT SOLAR RADIATION ON SUCCESSIVE LAYERS OF A CARBON CLOUD (N = 10,000 PARTICLES/cm^3)

(Cloud Height 200-100 mb) ($z = 0^o$)

Spectral Region	Wave Length (μ)	Incident Intensities (ly/min) Layer 1	Layer 2	Layer 3	Layer 4
1.	.28-.42	.213	.113	.062	.035
2.	.42-.50	.231	.129	.072	.041
3.	.50-.60	.263	.154	.091	.055
4.	.60-.70	.227	.141	.088	.055
5.	.70-.74	.077	.050	.033	.022
6.	.74-.79	.089	.059	.040	.027
7.	.79-.84	.075	.052	.035	.024
8.	.84-.86	.030	.021	.015	.010
9.	.86-.98	.143	.104	.075	.054
10.	.98-1.05	.069	.052	.039	.029
11.	1.05-1.22	.138	.107	.083	.064
12.	1.22-1.61	.181	.148	.122	.100
13.	1.61-2.10	.099	.086	.074	.064
14.	2.10-2.20	.013	.012	.010	.009
15.	2.20-3.00	.052	.047	.043	.039
16.	3.00-3.80	.020	.019	.017	.016
17.	3.80-4.50	.008	.008	.007	.007
18.	4.50-10.00	.012	.011	.011	.011
Median Wavelengths = $\bar{\lambda}$ =		.72μ	.79μ	.90μ	1.00μ

TABLE 11

INTENSITIES AND MEDIAN WAVELENGTHS OF INCIDENT SOLAR RADIATION ON SUCCESSIVE LAYERS OF A CARBON CLOUD (N = 20,000 $\frac{\text{particles}}{\text{cm}^3}$)
(Cloud Height 200-100 mb) (z = 0^{o})

Spectral Region	Wave Lengths (μ)	Incident Intensities (ly/min) Layer 1	Layer 2	Layer 3	Layer 4
1.	.28-.42	.213	.062	.022	.012
2.	.42-.50	.231	.072	.026	.012
3.	.50-.60	.263	.091	.034	.015
4.	.60-.70	.227	.088	.035	.016
5.	.70-.74	.077	.033	.014	.007
6.	.74-.79	.089	.040	.018	.008
7.	.79-.84	.075	.035	.017	.008
8.	.84-.89	.030	.015	.007	.004
9.	.89-.98	.143	.075	.040	.021
10.	.98-1.05	.069	.039	.022	.012
11.	1.05-1.22	.138	.083	.050	.030
12.	1.22-1.61	.181	.122	.082	.055
13.	1.61-2.10	.099	.074	.056	.042
14.	2.10-2.20	.013	.010	.008	.006
15.	2.20-3.00	.052	.043	.035	.029
16.	3.00-3.80	.020	.017	.015	.013
17.	3.80-4.50	.008	.007	.006	.006
18.	4.50-10.00	.012	.011	.010	.009
Median Wavelengths =		$\bar{\lambda}$ = .72μ	$\bar{\lambda}$ = .90μ	$\bar{\lambda}$ = 1.10μ	$\bar{\lambda}$ = 1.27μ

$\alpha = 1$ where $\alpha = \frac{2\pi r}{\lambda}$. Hence, a 39% increase in wavelength would require a 39% increase in particle radius to maintain optimum absorption efficiency. Since initial optimum particle size was determined to be about $r = .11\mu$, the optimum particles in the bottom layer of the model cloud would be about $r = .13\mu$ for $N = 4,000$ particles/cm^3 and $r = .15\mu$ for $N = 10,000$ particles/cm^3. Particle size variations of these magnitudes do not have a significant effect upon total cloud absorption values.

In addition lower overall light intensities in the lower layers make the importance of having particles of optimum size less in lower layers than in higher layers. Therefore, for carbon particle concentrations likely to be used in weather modification work, uniform particle sizes can be used without appreciable loss of absorption efficiency per unit mass.

Atmospheric Residence Times. It is desirable to know approximately how long dispersed carbon particles are likely to remain in the atmosphere. The principal removal mechanisms are rainout (particles becoming attached to raindrops during the condensation process) and washout (particles which are captured by raindrops during precipitation). Gravitational fallout may also affect clumps of particles should they coagulate to form particles with radii greater than 1μ, but this is not likely to be a significant factor. Martel (1970) estimates that natural atmospheric particles in middle latitudes have residence times of around 6 days in the lower and middle and 14-21 days in the upper

troposphere. Natural aerosol particles range from about .01 μ to 100 μ in size with the greatest number of particles being concentrated in the size range from .01μ to .1μ. Carbon particles of .1μ radius should not differ greatly from the mean atmospheric particles with respect to size. Although carbon particles differ in composition from most natural atmospheric aerosols, they should have aerodynamic properties similar to those of other basically inert particles. Therefore, an assumption that carbon particles dispersed in the low or middle tropical troposphere would have mean residence times of but 3 to 8 days seems reasonable. This indicates that carbon dispersed in an area with low ventilation (such as a closed circulation) could act as a heat source for more than one day. However, carbon dispersed in the tropical boundary layer would likely dissipate much faster due to the high levels of convective activity found there.

Major Findings. Computations of daily absorption of solar radiation by several configurations of carbon clouds were performed. Carbon black was shown to be a highly efficient absorber of short wave radiation.

The optimum particle size for absorption of solar radiation was determined to be $r = .11\,\mu$, but this exact radius is not extremely critical.

Absorption per unit mass of carbon decreases with increasing carbon concentrations due to the screening out of radiation in the upper layers of the cloud. This effect greatly reduces absorption efficiency at carbon horizontal area coverages greater than about 25%.

Clouds with low concentrations of carbon are relatively efficient absorbers throughout the solar day, while high density clouds are efficient absorbers per unit mass only when the solar zenith angle is small.

Redundant absorption tendencies of carbon and water vapor cause some loss of efficiency in carbon cloud absorption compared to absorption of the clear surrounding air. However, this effect is relatively small (0-12% loss) in most of the cloud configurations tested. This efficiency loss is negligable for carbon clouds in the tropical boundary layer.

The longwave radiation loss of a carbon cloud is not significantly greater than the longwave radiation loss by an equivalent clear air mass. Therefore, longwave flux divergence should not cause a heat loss by the carbon cloud relative to the surrounding air.

Virtually 100% of the solar radiation absorbed by the carbon particles is transmitted as heat to the surrounding air.

Decreasing surface albedos can increase the absorption of carbon clouds, but the net gain is relatively small (a cloud over a 50% albedo surface absorbs only 10-15% more radiation than a cloud over a 0% albedo surface).

In a vertically homogeneous cloud most of the absorption occurs in the upper layers of the cloud. To obtain uniform vertical absorption the carbon must be distributed with concentrations increasing towards the bottom of the cloud.

Although the median wavelengths of radiation incident on the lowest layers of the clouds are greater than that of incident solar radiation at the cloud tops, the wavelength shifts are not great enough to require the use of larger carbon particles in the lower cloud layers to maintian maximum absorption efficiency.

Carbon particles dispersed in the troposphere should exhibit residence times of approximately 3-8 days. This would indicate that it may be possible to obtain heat for more than one day in cases where the carbon cloud is located in areas of weak ventilation.

IV. CONCLUSIONS

Carbon Black as an Atmospheric Heat Source. The characteristic of carbon black which makes it attractive as an atmospheric heat source is the extraordinarily large amount of solar radiation per unit mass of carbon which can be absorbed and hence transmitted to the air. One gram of carbon can absorb more than 40 million calories of solar radiation in a single day. On the other hand coal, currently the cheapest of conventional combustable fuels, provides only about 7,000 cal per gram. The relative costs of heat available from carbon black and coal are shown in Table 12. The cost of coal heat is no less than 200 times greater than the cost of carbon heat. This figure becomes even higher if multiple day use of carbon is considered.

It is important to realize the amount of air temperature increase which is possible using carbon black. For example, in the tropical boundary layer (1013 - 950 mb), one C-5A aircraft carrying $\sim 1 \times 10^5$ Kg of carbon black could disperse a cloud covering over 4000 square kilometers and extending from sea level to 950 mb with a horizontal area coverage of 9%. This cloud could provide enough heat to increase the temperature of the air within the seeded boundaries by about $8^{\circ}C$ per 10 hours. Table 13 shows several cloud configurations which could be dispersed from one U. S. Air Force C-5A aircraft (Lockheed-Georgia Co., 1966) together with possible cloud heating rates.

Among energy sources normally used by man only nuclear energy compares with carbon black as a source of energy per unit mass, and

TABLE 12

RELATIVE COSTS OF COAL HEAT AND CARBON BLACK HEAT

Fuel	Cost (dollar/kg)	Heat ($\frac{cal}{kg}$)	Heat per Unit Cost ($\frac{cal}{dollar}$)
Coal	\$.006	$7 \times 10^6 \frac{cal}{kg}$	$1.2 \times 10^9 \frac{cal}{dollar}$
Carbon Black	\$.15	$4 \times 10^{10} \frac{cal}{kg}$	$2.7 \times 10^{11} \frac{cal}{dollar}$
Ratio $\frac{(Carbon\ Black)}{Coal}$	$\frac{25}{1}$	$\sim \frac{6000}{1}$	$\sim \frac{200}{1}$

no known substance compares as a source of heat per unit cost. A 20 Kiloton nuclear explosion produces about the same amount of thermal energy that 1,000 kg of carbon black produces in 10 hours. The carbon is not consumed during the heat generation process, and if atmospheric conditions are chosen carefully, residence times of at least several days are possible with resulting increases in efficiency. The major expense to be expected when attempting to utilize carbon black as an atmospheric heat source is the cost of air dispersal. Preliminary cost analysis (below) indicates that the cost of air dispersal of large amounts of carbon black from aircraft, such as the U.S. Air Force C-5A, would be 2-3 times the price of the carbon black itself. Even so, the available heat per unit cost is very large.

Preliminary Cost Analysis of Air Dispersal of Carbon Black Particles. It is assumed that the amount of carbon black which is to be dispersed will be greater than 1×10^5 kg. This minimum amount is approximately the amount of carbon black which might be dispersed by

TABLE 13

TYPICAL TROPICAL BOUNDARY LAYER CARBON DUST INDUCED HEATING RATES

Total carbon mass = 1×10^5 kg (1 C-5A full load)

Cloud Height = 0.55 km

Cloud Configuration: Circular (radius = r)

Area	% Area Coverage	Net Heat Absorbed $(\frac{cal}{cm^2 10hrs.})$	Temperature Change (°C/10hrs.)
$7200km^2$	5%	60	4
$4000km^2$	9%	110	8
$2000km^2$	18%	200	13
$1500km^2$	26%	270	18
$1000km^2$	35%	325	22
$680km^2$	53%	410	27
$510km^2$	70%	470	31

one C-5A during one mission. The C-5A is the most economical aircraft in service at the present time with respect to pay-load-miles per unit cost. The cost of dispersal, therefore, will approximately equal the cost of operating a C-5A for 1 mission.

Based on atmospheric diffusion estimates by Turner (1969), particles dispersed from a C-5A can be assumed to diffuse at least 1 km in distance from the flight path in a few minutes to an hour. Flying at 400 knots a C-5A could disperse a carbon dust cloud of $\sim 1000\,km^2$ horizontal area per flight hour. For many of the large scale weather modification cloud dust models envisaged, optimum carbon concentrations from about 5% to 20% horizontal area coverage are desirable.

Choosing a representative horizontal area coverage of 12%, a C-5A flying at 400 knots could cover an area of about 4,000 km^2 with a carbon cloud of that density in a 3 hour period. Allowing a total of 2-3 hours for flight time to and from the seeding area, an approximate standard mission of 8-10 hours duration may be determined. It is assumed, therefore, that the cost of dispersal will be roughly the cost of operating a C-5A for 8-10 hours. This standard mission assumes that the carbon cloud is to be concentrated in a relatively shallow vertical layer (0 - 2 km) such as the boundary layer cloud for possible tropical storm modification. A thick carbon cloud, such as one encompassing the entire vertical extent of the tropopause, would require much more flight time and higher costs.

It is assumed that weather modification experiments and operations of the magnitude considered here would be government sponsored at least in the initial stages. Due to the varied and fluctuating funding procedures utilized by the government at this time, it is impossible to determine total actual costs before an administrative organization has been established. The figures below are based on March 1972 price levels as supplied by the Air Force Flight Test Center, Edwards AFB, California.

C-5A Operating Costs ($ dollars/hr)	
Consumables:	$1000 - $2000
Support:	$1000 - $2000
Crew:	$100
TOTAL	$2100 - $4100

The best guess is that total operating costs would be approximately $3,000/hr. For one 8-hour mission:

C-5A Operating Costs (8hrs):	$25,000
1×10^5 kg carbon black $0.15/kg:	15,000
TOTAL	$40,000

Neglecting project expenses and hardware development costs, the cost of seeding clear air with 1×10^5 kg of carbon black would average approximately $40,000. This figure could vary downward by nearly a factor of 2 if the operation was carried out within the operating structure of the Department of Defense or upward by a similar amount if the project was carried out by an organization with a less favorable inventory structure.

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REFERENCES

Cabot, G. L., Inc., (1949): Carbon Black, Encyclopedia of Chemical Technology, Vol. 3, pp. 34-65.

Chandrasekhar, S., (1960): "Radiative Transfer", Dover Publications Inc., New York, 393pp.

Dave, J. V., and P. M. Furukawa, (1966): "Scattered Radiation in the Ozone Absorption Bands at Selected Levels of a Terrestrial, Rayleigh Atmosphere", Meteorological Monographs, Vol. 7, No. 29, 270pp.

Downie, Major C. S., and R. B. Smith, (1958): "Thermal Techniques for Dissipating Fog from Aircraft Runways", Air Force Surveys in Geophysics No. 106, Air Force Cambridge Research Center, Bedford, Massachusetts, 38pp.

Dushman, S., (1949): "Scientific Foundations of Vacuum Technique", John Wiley and Sons, Inc., New York, 547pp.

Fenn, R. W., (1964): "Theoretical Aspects of Carbon Seeding", U. S. Army Electronics Research and Development Laboratories Report, 18pp.

_____, and H. Oser, (1962): "Theoretical Considerations on the Effectiveness of Carbon Seeding", U. S. Army Signal Research and Development Laboratory Report No. 2258, 25pp.

_____, (1965): "Scattering Properties of Concentric Soot-Water Spheres for Visible and Infrared Light", Applied Optics, Vol. 4, No. 11, pp. 1504-1509.

Korb, Gunther, and Fritz Möller, (1962): "Theoretical Investigation on Energy Gain by Absorption of Solar Radiation in Clouds", Final Report on U. S. Army Signal Corp Contract No. DA-91-541-EUC-1612, 185pp.

Lettau, H. H., (1965): data published by William D. Sellers, "Physical Climatology", University of Chicago Press, Chicago, 272pp.

Lockheed Georgia Corp., (1966): "Introducing the Lockheed U. S. Air Force C-5A," Internal Company Publication, 49pp.

REFERENCES (cont'd)

Martell, E. A., (1970): "Transport Patterns and Residence Times for Atmospheric Trace Constituents vs. Altitude", Advances in Chemistry Series, 93, American Chemical Co. Washington, pp. 421-428.

Smith, R. B., R. Wexler, and A. H. Glaser, (1959): "Modification of Fog and Cloud Particles in the Atmosphere", Allied Research Associates Inc., Final Report on Contract No. AF 19 (604)-3492, 47pp.

Turner, D. B., (1969): "Workbook of Atmospheric Estimates", Public Health Service Publication No. 999-AP-26, 84pp.

van Straten, F. W., R. E. Ruskin, J. E. Dinger, and H. J. Mastenbrook, (1958): "Preliminary Experiments Using Carbon Black for Cloud Modification and Formation", U. S. Naval Research Laboratory Report No. 5235, 17pp.

Wexler, R., (1958): "Seeding with Carbon Black", Allied Research Associates Inc. Documents No. ARA-553, 7pp.

APPENDIX I.

Equations for absorption, reflection, and transmission of incident solar radiation, derived by Korb and Möeller[4] from Chandrasehkar's equation of radiative transfer.

A) for albedo = 0.

Transmission = T_o

Reflection = R_o

Absorption = A_o

Zenith angle = z

$$T_o = \frac{1}{2\cos z}\left[\frac{(-M+N)\left[e^{-(\sec z-\sqrt{EF})t}-e^{-(\sec z+\sqrt{EF})t}\right]+(M+N)\left(\frac{P}{Q}-\frac{Q}{P}\right)}{\left(\frac{Q}{P}\right)e^{\sqrt{EF}\,t}-\left(\frac{P}{Q}\right)e^{-\sqrt{EF}\,t}}\right] + e^{-(\sec z)t}\left(1+\frac{M+N}{2\cos z}\right) \quad (A1)$$

$$R_o = \frac{1}{2\cos z}\left[\frac{(-M+N)\left(\frac{Q}{P}-\frac{P}{Q}\right)e^{-t\sec z}+(M+N)\left[e^{-t\sqrt{EF}}-e^{+t\sqrt{EF}}\right]}{\left(\frac{Q}{P}\right)e^{t\sqrt{EF}}-\left(\frac{P}{Q}\right)e^{-t\sqrt{EF}}}\right] \quad (A2)$$

$$A_o = 1 - T_o - R_o \quad (A3)$$

K_a = absorption Quantity

Where:

$$E = (\alpha_o - \beta_o)(1 - K_A) - 1 \quad (A4)$$

$$F = (\alpha_o + \beta_o)(1-K_A) - 1 + \frac{2\alpha_{90}(1-K_A)^2\gamma_z}{1-\gamma_{90}(1-K_A)} \quad (A5)$$

$$G = (\alpha_z - \beta_z)(1-K_A) \quad (A6)$$

$$H = (\alpha_z + \beta_z)(1-K_A) + \frac{2\alpha_{90}(1-K_A)^2\gamma_z}{1-\gamma_{90}(1-K_A)} \quad (A7)$$

$$K = EH - G \sec z \tag{A8}$$

$$L = FG - H \sec z \tag{A9}$$

$$M = \frac{K}{\sec^2 z - EF} \tag{A10}$$

$$N = \frac{L}{\sec^2 z - EF} \tag{A11}$$

$$P = \sqrt{EF} + F \tag{A12}$$

$$Q = \sqrt{EF} - F \tag{A13}$$

$$t = \text{optical depth} = \int_0^h (\sigma_E N\pi r^2 + \rho_W K_W)\, dh \tag{A14}$$

N = number particles/cm^3

r = radius particle = .1μ

ρ_W = density water

K_W = absorption quantity of water vapor

α, β, γ are scattering coefficients and depend upon the zenith angle of the sun. Values used are:

	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
α	.325	.320	.313	.305	.295	.282	.265	.244	.218	.184
β	.108	.111	.115	.119	.124	.129	.140	.152	.165	.184
γ	.567	.569	.572	.576	.581	.589	.595	.604	.617	.632

B) For albedo $\neq$ 0

T_T = Transmitted

R_T = Reflected

A_T = Absorbed

$$T_T = \frac{T_o}{1 - q_1} \text{ where} \tag{A15}$$

$$q_1 = \frac{A_s}{2}\left[\frac{(N'-M')\ (Q/P - P/Q)e^{-t} + (M'-N')\ (e^{-t\sqrt{EF}} - e^{+t\sqrt{EF}})}{(Q/P)e^{t\sqrt{EF}} - (P/Q)\ e^{-t\sqrt{EF}}} \right. \quad \text{(A16)}$$

$$\left. -(N'-M') \right]$$

$$\text{and } R_T = R_o + q_2\ T_T \quad \text{(A17)}$$

where

$$q_2 = \frac{A_s}{2}\left[\frac{(N'-M')\ [e^{-(t-\sqrt{EF}\ t)} - e^{-(t+t\sqrt{EF})}] + (M'+N')\ (P/Q - Q/P)}{(\frac{Q}{P})\ e^{t\sqrt{EF}} - (\frac{P}{Q})\ e^{-t\sqrt{EF}}} \right. \quad \text{(A18)}$$

$$\left. + e^{-t}(2+M'+N') \right]$$

$$G' = (\bar{\alpha} - \bar{\beta})\ (1 - K_A) \quad \text{(A19)}$$

$$H' = (\bar{\alpha} + \bar{\beta})\ (1-K_A) + \frac{2\alpha_{90}(1-K_A)^2\bar{\gamma}}{1-\gamma_{90}(1-K_A)} \quad \text{(A20)}$$

$$K' = EH' - G' \quad \text{(A21)}$$

$$L' = FG' - H' \quad \text{(A22)}$$

$$M' = \frac{K'}{1-EF} \quad \text{(A23)}$$

$$N' = \frac{L'}{1-EF} \quad \text{(A24)}$$

A_s = albedo expressed as a percentage

$\bar{\alpha}$ = .275

$\bar{\beta}$ = .135 mean scattering coefficients

$\bar{\gamma}$ = .590